

A CMB/Dark Energy Cosmic Duality

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We investigate a possible connection between the suppression of the power at low multipoles in the CMB spectrum and the late time acceleration. We show that, assuming a cosmic IR/UV duality between the UV cutoff and a global infrared cutoff given by the size of the future event horizon, the equation of state of the dark energy can be related to the apparent cutoff in the CMB spectrum. The present limits on the equation of state of dark energy are shown to imply an IR cutoff in the CMB multipole interval of $9 > l > 8.5$.

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While the Λ CDM model describes the CMB angular spectrum well, WMAP data [1] indicates that there could be a problem at largest scales. Compared with the model predictions, at low multipoles there appears to be a lack of power; according to simulations, the observed WMAP quadrupole has a low probability [1]. Of course, the explanation might simply be cosmic variance combined with bad luck, but there is also the possibility that the low multipoles signal for new physics. Effectively, reducing the angular power at low multipoles requires an introduction of an cutoff in the infrared. Such cutoff could be dynamical [2] and due to, say, properties of the inflaton potential such that the slope of the potential changes much during the beginning of the last 65 e-folds. A period during which the inflaton kinetic energy dominates could also be one possible explanation. The common feature of these scenarios is that they rely on local physics, and that tuning of the parameters is necessary.

Another, although admittedly a more speculative possibility, is that there exist global constraints which manifest themselves as an IR cutoff. An example is the holographic principle [3], which states that the field theoretical description overcounts the true dynamical degrees of freedom and hence, if true, one should impose extra, non-local constraints on the effective field theory. Indeed, it is well known that a local quantum field theory confined to a box of size \tilde{L} and having a UV cutoff Λ can not fully describe black holes while preserving unitarity. For any Λ there is a sufficiently large volume for which the entropy of an effective field theory will violate the Bekenstein bound [4]; the field theory overcounts the true dynamical degrees of freedom. The same conclusion holds also for quantum gravity: local quantum field theory is not a good description of gravity because it has too many degrees of freedom in the UV.

The Bekenstein entropy bound may, however, be sat-

isfied in an effective field theory if we limit the volume of the system according to

$$\tilde{L}^3 \Lambda^3 \leq S_{BH} = \pi \tilde{L}^2 M_P^2 \quad (1)$$

where S_{BH} is the entropy of a black hole of radius \tilde{L} . Thus, the length scale \tilde{L} , which provides an IR cutoff, is determined by the UV cutoff Λ and can not be chosen independently.

As pointed out by Cohen et al. [5] (see also [6]), the actual bound on the volume could be even stronger. An effective field theory that can saturate Eq. (1) includes many states with a Schwarzschild radius much larger than the box size. It seems plausible that we should exclude such states from the effective field theoretical description. This lead Cohen et al. to propose an additional constraint on the IR cutoff. Since the maximal energy density in the effective theory is Λ^4 , requiring the energy in a given volume not to exceed the energy of a black hole of same size results in the constraint [5]

$$\tilde{L}^3 \Lambda^4 \lesssim \tilde{L} M_P^2. \quad (2)$$

It is interesting to observe that if one takes \tilde{L} to be the size of the observable universe today i.e. the current particle horizon, given approximately by the Hubble scale H_0^{-1} , saturating Eq. (2) one obtains a vacuum energy density of the right order of magnitude, consistent with observations [5, 6, 7, 8]. This is an interesting observation since the size of the cosmological constant is one of the great finetuning problems in cosmology and also lies at the core of the cosmic coincidence problem. Unfortunately, it has been shown that the resulting equation of state for dark energy does not agree with data [7]. However, as suggested by Li [8] (see also [9, 10]), it might be more natural to take the IR cutoff to be the size of the volume a given observer eventually will observe. This is the future event horizon R_H , given by

$$R_H = a \int_t^\infty \frac{dt}{a}. \quad (3)$$

Thus, a local field theory should describe only the degrees of freedom that can ever be observed by a given observer.

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In this respect the observer is always in a privileged position. Note that the future event horizon depends on the equation of state. This approach could also be in accordance with the black hole complementarity conjecture [11] (for a discussion of de Sitter complementarity see [12]), although the way holography manifests itself is likely to be more convoluted [13]. However, for phenomenological purposes it is interesting to assume that the same IR cutoff that is responsible for the dark energy viewed as a quantum fluctuation whose size is determined by the IR/UV duality, is also present in the spectrum of CMB perturbations. Thus, in effect we assume that the same nonlocal, possibly holographic theory which at the quantum level ensures that there is no overcounting of degrees of freedom, at the classical level is also relevant for the CMB anisotropies. Also, by virtue of Occam's razor, we could claim that if we observe an IR cutoff in the CMB, the most economic assumption is that it is the same IR cutoff which is responsible for the small effective cosmological constant or dark energy component.

Hence our starting point is the following. Let ρ_Λ be the quantum zero-point energy density rendered finite by the UV cutoff. The total energy in a region of spatial size \tilde{L} should not exceed the mass of a black hole of the same size, or $\frac{4\pi}{3}\tilde{L}^3\rho_\Lambda \leq 4\pi\tilde{L}M_P^2$. The largest region \tilde{L} allowed is the one saturating this inequality so that we may write

$$\rho_\Lambda = 3M_P^2\tilde{L}^{-2} = 3c^2M_P^2R_H^{-2} . \quad (4)$$

Following Li [8] we have introduced the constant c , but here its physical meaning is slightly different since here we have related the IR cutoff \tilde{L} and the future event horizon R_H by $R_H \equiv c\tilde{L}$. They should be of same order but could differ by some factor which in principle would be calculable in the full theory. Hence we expect that $c \sim \mathcal{O}(1)$. For instance, in a generalization of Holographic dark energy to curved space [9] it has been proposed that $\tilde{L} = a \sin(y)/\sqrt{k}$ where $y = \sqrt{k}R_h/a$. This would effectively lead to $c \approx 1/\cos(\sqrt{k}R_h) > 1$ [9]. However, here we restrict ourselves to flat space and consider c as a free parameter.

To translate \tilde{L} into an IR cutoff at physical wavelengths we can simply consider quantization in a spherical potential well with infinitely high potential walls. The radial solutions are spherical Bessel functions with the ground state j_0 being proportional to $\sin(kr)/kr$ with $r < r_a$. Here r_a is the radius on the spherical potential. Demanding that the solution vanishes at $r = r_a$ one finds that the wavelength of the ground state is $\lambda_c = 2r_a$. Thus the IR cutoff \tilde{L} translates into a cutoff at physical wavelengths given by $\lambda_c = 2\tilde{L}$.

Using the Friedman equation it can be shown that for a cosmic two component fluid comprising of a matter component and a dark energy component Ω_Λ , in a flat

universe Eq. (4) implies that

$$R_H = a^{3/2}c \frac{1}{\sqrt{\Omega_m^0}H_0} \left(\frac{1 - \Omega_\Lambda}{\Omega_\Lambda} \right)^{1/2} .$$

which means that, assuming a flat universe, at present time we have

$$R_H = \frac{c}{\sqrt{\Omega_\Lambda^0}}H_0^{-1} . \quad (5)$$

Let us first point out that in the CMB angular spectrum the IR cutoff would show up in the Sachs-Wolfe effect

$$C_l = \pi \int_0^\infty \frac{dk}{k} j_l^2(k\eta_0) \delta_H^2(k) ,$$

which is an integral over all comoving momentum modes. The IR cutoff would result in a lower bound on the comoving momentum and also turn the integral into a sum over the discrete momentum modes, hence suppressing the Sachs-Wolfe contribution and CMB spectrum at small l s, so symbolically we can then write the Sachs-Wolfe effect as

$$C_l \simeq N \sum_{k > k_c} \frac{1}{k} j_l^2(k\eta_0) \delta_H^2(k) . \quad (6)$$

Here k_c is the IR comoving momentum mode cutoff, $\delta_H^2(k) \equiv \frac{2}{25}\mathcal{P}_\mathcal{R}(k)$ is the primordial power spectrum, N a normalization factor, and j_l is the spherical Bessel function. If the spectrum is a single power law with a spectral index n , we have $\delta_H^2(k) \propto k^{n-1}$. Note that the spacing in the sum in Eq. (6) is not equidistant and the discrete spectrum approaches rapidly the continuum.

This type of cutoff looks much less like a step function than the type of phenomenological cutoffs associated with inflaton dynamics discussed in the literature [2, 14]. Thus we expect that the IR cutoff due to UV/IR duality must lie at larger length scales than the usual best fit. In addition, because of the discreteness, our approach will give rise to oscillatory features in the low l spectrum. Here it may be noted that the WMAP team actually considered a toy model with a discrete equidistant spectrum and suggested that it might fit the data better than the conventional power spectrum [1]. We should also mention that since the setup here is different, we do not expect the geometric patterns that usually appear in models of finite universes realized as multi-connected spaces [15]. In Fig. 1 we show the Sachs-Wolfe effect for a model with a large scale suppression due to an early epoch with a blue spectrum, compared to the type of cutoff suggested here.

In flat space the multipole number l is given by $l = k_l(\eta_0 - \eta_*)$ where $\eta_0 - \eta_*$ is the comoving distance to the last-scattering surface and k_l is the corresponding comoving wave number. The comoving distance to last scattering follows from the definition of comoving time

$$\eta_0 - \eta_* = \int_0^{z_*} dz' \frac{1}{H(z')} . \quad (7)$$

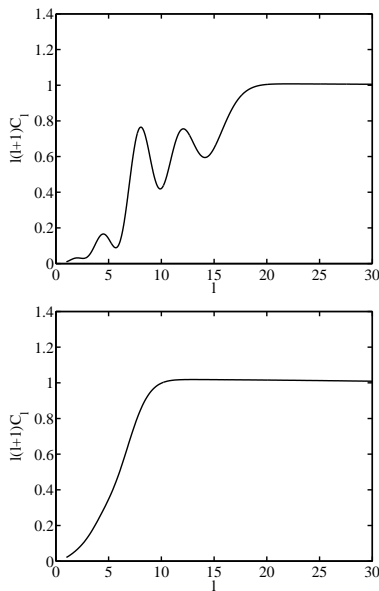


FIG. 1: Upper panel: the Sachs-Wolfe effect for a spectrum with a sharp cutoff at $l = 10$ due to UV/IR duality. Lower panel: the Sachs-Wolfe effect for a spectrum with a blue $n = 4$ tilt for $l < 10$ and a flat $n = 1$ spectrum for $l > 10$.

Assuming that the dominant components of energy in the universe are dark energy and matter, then

$$H^2(z) = H_0^2 \left[\Omega_\Lambda^0 (1+z)^{(3+3w_\Lambda)} + (1 - \Omega_\Lambda^0)(1+z)^3 \right],$$

where w_Λ is given by the equation of state of dark energy $p_\Lambda = w_\Lambda \rho_\Lambda$. If w_Λ is approximated by a constant, we can find an analytical solution for Eq. (7) in terms of the Gauss hypergeometric function

$$\eta_0 - \eta_* \simeq \frac{2}{\sqrt{\Omega_m^0} H_0} {}_2F_1 \left(\frac{1}{2}, -\frac{1}{6w_\Lambda}; 1 - \frac{1}{6w_\Lambda}; -\frac{\Omega_\Lambda^0}{\Omega_m^0} \right) \quad (8)$$

In Fig. 2 we show a plot of the solution to Eq. (8). However, one should note that we have slightly overestimated $\eta_0 - \eta_*$ by treating w_Λ as a constant. An exact numerical solution is also shown in Fig. 2.

On the other hand, the equation of state of dark energy can be shown to be related to the constant c by $w_\Lambda^0 = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda^0}$ [8]. Thus, the distance to last scattering is a function of w_Λ which we can fit by fixing the free parameter c . However, from Eq. (5) we see that \tilde{L} does not depend on c . Using $\Omega_\Lambda^0 = 0.75$ and $\Omega_m^0 = 0.25$ we find $\tilde{L} = 1.2 \times H_0^{-1}$ today, which implies $\lambda_c = 2\tilde{L} = 2.4 \times H_0^{-1}$ or $1/k_c \equiv \lambda_c/(2\pi) = 1.2/\pi \times H_0^{-1}$. Since the scale corresponding to the first multipole in the CMB spectrum is the distance to last scattering, we see that the position of a large scale cutoff \tilde{L} relative to the first multipole has a weak dependence on the equation of state of the dark energy.

We are interested in the case where the cutoff is in the fiducial part of the spectrum. For the suppression of the low multipole to be statistically significant, we probably need to suppress at least the two first multipoles in the CMB spectrum corresponding to $l = 2, 3$, i.e. $1/k_c \lesssim 1/k_{l=3} = (\eta_0 - \eta_*)/3$. Using $1/k_c = 1.2/\pi \times H_0^{-1}$ and Eq. (8), we find that the cutoff is indeed in the interesting range [18]. Combining data from WMAP and

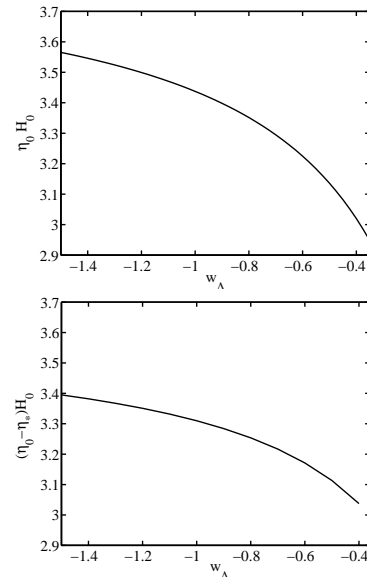


FIG. 2: Upper panel: The present distance to last scattering, η_0 , as a function of the equation of state today w_Λ^0 , given by Eq. (8). Lower panel: A numerical solution where the proper redshift dependence of w_Λ has been taken into account (see also Fig. 3).

other cosmic microwave background experiments with large scale structure, supernova and Hubble Space Telescope data, the dark energy equation of state has been found [16] to be limited by $-1.38 < w_\Lambda < -0.82$ at the 95% confidence level. Given these experimental limits, the position of the cutoff l_c in the multipole space falls in the interval $8.5 < l_c < 9$. This is probably a bit larger than the preferred value [2] which corresponds to $l_c \approx 7$, but still is comfortably within the 2σ limit corresponding to $l_c \approx 10$; however, these numbers are just suggestive as a global fit would be needed to find the exact position and confidence levels of l_c .

The scenario presented here can be viewed as a toy model of cosmic duality where the IR cutoff plays an important role. It is worth mentioning that in a universe dominated by dark energy we actually live inside a finite box, the so-called "causal diamond" of the static de Sitter coordinates, which is bounded by the past and future event horizons [12]. In cosmic coordinates the finiteness could manifest itself as an effective IR regulator of the same order of magnitude as the future event horizon, which in a pure de Sitter space determines also the mag-

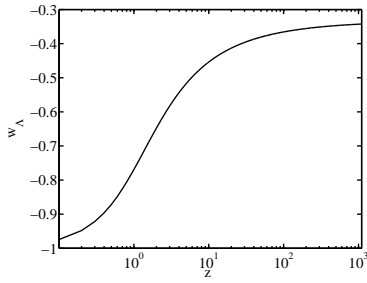


FIG. 3: The equation of state of dark energy w_Λ as a function of redshift z for the specific case of $w_\Lambda^0 = -1$ and $c = 0.87$.

nitude of the effective cosmological constant. Of course, even if an IR/UV duality is at work in the theory at some fundamental level, the IR regulator might not be simply related to the future event horizon but there might still be a (complicated) relation between the dark energy and the IR cutoff of the CMB perturbation modes.

In passing, it is interesting to note that in [17] it is shown that the future event horizon also limits the number of e-foldings of inflation that will ever be observable in the CMB spectrum.

We conclude that a relation between the location of the cutoff in the CMB spectrum and the equation of state of dark energy is something that can be searched for in the CMB data. However, a detailed global fit to data, which would simultaneously fix the value of the cutoff l_c and all the cosmological parameters, is beyond the scope of this paper, where we merely point out to a possible interesting connection. For the purposes of falsifying the cosmic IR/UV duality proposed here, the redshift dependence of w_Λ (see Fig 3.), the exact location of the cutoff l_c and the predicted shape of the spectrum at low l would all be crucial. Here $w_\Lambda(z)$ is not an independent quantity but depends on the CMB IR cutoff l_c and the amount of dark energy. As the time evolution of the equation of state of dark energy may be detectable too in the near future, the present scheme can also be put to test.

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